1

Lesson 19: Process Characteristics-1st Order Lag & Dead-Time Processes

ET 438a Automatic Control Systems Technology

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2

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Learning Objectives

After this series of presentations you will be able to:

- Describe typical 1st order lag and dead-time process models found in control systems.
- Write mathematical formulas for 1st order and dead-time process models
- Compute the parameters of process models.
- Identify the Bode plots of typical process models.
- Identify the time response of typical process models.

First Order Lag Process

Characteristics:

1) Single storage element
2) Input produces an output related to amount of storage
3) Another name: self-regulating process

Examples:

Series R-C circuit
Series R-L circuit
Self-regulating tank (valve on output)
Tank heating

First Order Lag Process

Mathematical Descriptions

Time domain equation: $\tau \cdot \left(\frac{dy}{dx}\right) + y = G \cdot x$

Transfer function: $\frac{Y(s)}{X(s)} = \frac{G}{1+\tau \cdot s}$

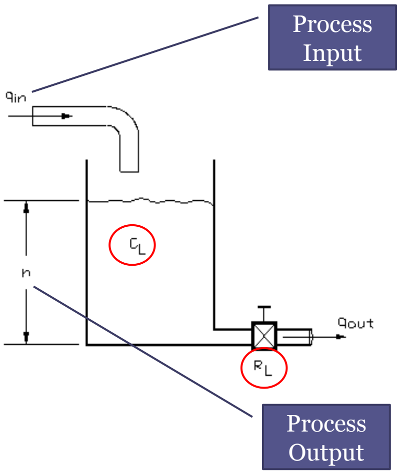
Equation Constants: Dependent on System

τ determines time required for system to reach final value after step input

1τ= 63.2% Final value
5τ=99.3% Final value

Where: G = steady-state gain of the system
t = time, in seconds
y = output of process (units or %FS)
x = input of process (units or %FS)
τ = time constant of the system in seconds

First Order Lag Process – Self-Regulating Tank



Process input: q_{in} = input flow rate (%FS_{in})
FS_{in} = input range (m³/s)
Process output: h = tank level, (% FS_{out})
FS_{out} = output range (m)

<u>Process Parameters</u>
R_L = flow resistance
C_L is the tank capacitance

Equation Constants

$$\tau = R_L \cdot C_L$$

$$G = \frac{R_L}{\rho \cdot g} \cdot \left[\frac{FS_{in}}{FS_{iout}}\right]$$

Where:
ρ = liquid density (kg/m³)
g = acceleration due to gravity 9.81 m/s²

First Order Lag Process Self-Regulating Tank

<u>Example 19-1:</u> Oil tank similar to previous figure has a diameter of 1.25 m and a height of 2.8 m. The outlet pipe is a smooth tube with a length of 5 m and diameter of 2.85 cm. Oil temperature 15 degrees C. The full scale flow rate is 24 L/min and full scale height is 2.8 m. Determine:

a) tank capacitance, C_L
b) pipe resistance, R_L
c) process time constant, τ
d) process gain, G
e) time-domain equation
f) transfer function.

Example 19-1 Solution (1)

a) Find the tank capacitance

$$C_L = \frac{A}{\rho g} \qquad d = 1.25\ m \qquad \rho = 880\ kg/m^3$$

From Appendix A

$$A = \frac{\pi d^2}{4} = \frac{\pi (1.25 m)^2}{4} = 1.23\ m^2$$

$$C_L = \left[\frac{1.23 m^2}{(880 kg/m^3)(9.81 m/s^2)}\right] = 1.425 \times 10^{-4}\ m^3/Pa$$

Example 19-1 Solution (2)

b) Find the flow resistance. First compute the Reynolds number to determine flow type. Use maximum flow to determine it.

$$R = \frac{\rho v d}{\mu} \qquad v = \text{average velocity} \quad \mu = \text{absolute viscosity} \qquad Q = 24\ L/min$$

Convert to m³/s

$$Q = 24\ L/min \left(1.6667 \times 10^{-5} \frac{m^3/s}{L/m}\right) = 4 \times 10^{-4}\ m^3/s$$

$$Q = VA \qquad A_p = \text{Area of Pipe}$$

Need pipe diameter for its area calculation

$$d_p = 2.85 cm$$

$$d_p = 2.85 cm \left[\frac{1 m}{100 cm}\right]$$

$$d_p = 2.85 \times 10^{-2}\ m$$

9

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Example 19-1 Solution (3)

$$A_p = \frac{\pi(2.85\times10^{-2}\,m)^2}{4}$$

$$A_p = 6.379\times10^{-4}\,m^2$$

$$V = \frac{Q}{A} = \frac{4\times10^{-4}\,m^3/s}{6.379\times10^{-4}\,m^2}$$

$$V = 0.627\,m/s$$

Now compute the Reynolds number

$$\mu = 0.160\ Pa\text{-}s$$

$$\rho = 880\ Kg/m^3$$

$$R = \frac{(880\,Kg/m^3)(0.627\,m/s)(2.85\times10^{-2}\,m)}{0.160\ Pa\text{-}s}$$

$$R = \frac{15.725}{0.160} = 98.3$$

Laminar Flow $R<2000$

10

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Example 19-1 Solution (4)

For Laminar flow

$$R_L = \frac{128\,\mu\,l}{\pi d^4}\ Pa\text{-}s/m^3$$

l = pipe length

$\mu = 0.160$ Pa-s

$$R_L = \frac{128(0.160\,P\text{-}s)(5\,m)}{\pi(2.85\times10^{-2}\,m)^4}$$

$$R_L = \frac{102.4}{\pi(6.5975\times10^{-7})} = \boxed{4.94\times10^{7}\ Pa\text{-}s/m^3}$$

Now compute the tank time constant

$$\tau = R_L C_L$$

$$\tau = (4.94\times10^{7}\,Pa\text{-}s/m^3)(1.425\times10^{-4}\,m^3/Pa)$$

$$\boxed{\tau = 7040\ s}$$

$$\tau = 7040\,s\left[\frac{1\ min}{60\,s}\right] = \boxed{117.3\ min}$$

Tank level reduced to 63.2% of initial value after 117.3 minutes with $q_{in}=0$. 99.2% empty after 5τ.

Example 19-1 Solution (5)

d) Compute process gain, G

$$G = \frac{R_L}{\rho g}\left[\frac{FS_{in}}{FS_{out}}\right]$$

$FS_{in} = 4\times10^{-4}\ m^3/s$

$FS_{out} = 2.8\ m$

$\rho = 880\ Kg/m^3$

$R_L = 4.97\times10^{7}\ Pa\text{-}s/m^3$

$$G = \frac{4.97\times10^{7}\ Pa\text{-}s/m^3}{(880\ Kg/m^3)(9.81\ m/s^2)}\left[\frac{4\times10^{-4}\ m^3/s}{2.8\ m}\right]$$

$$G = 5722.4\,(1.429\times10^{-4}) = \boxed{0.818}$$

Ans

Example 19-1 Solution (6)

e) Find the differential equation for this process

General Equation

$$\tau \frac{dy}{dt} + y = Gx$$

$x = q_{in}$ (inflow)

$y = h$ (Tank liquid Level)

q_{in} and h are in % FS

$\tau = 7040\ s$

$G = 0.818$

$$\boxed{7040\frac{dh}{dt} + h = 0.818\,q_{in}}$$

Ans

f) Find the transfer function for this process

General Equation

$$\frac{H(s)}{Q_{in}(s)} = \frac{G}{1+\tau s} = \boxed{\frac{0.818}{1+7040s}}$$

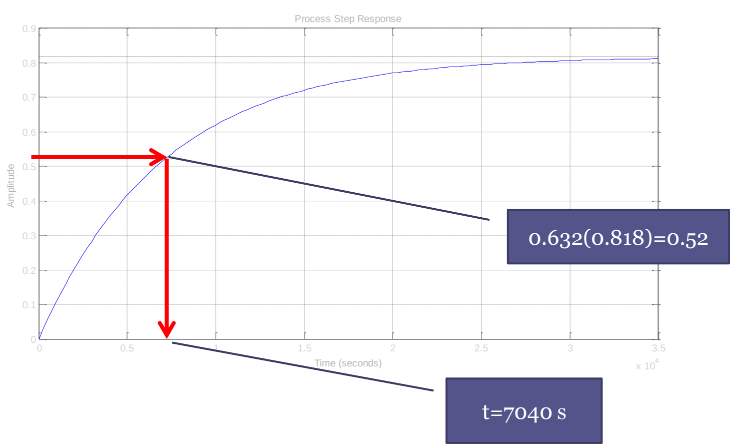
Ans

Step Response and Bode Plots of The First-Order Lag Process

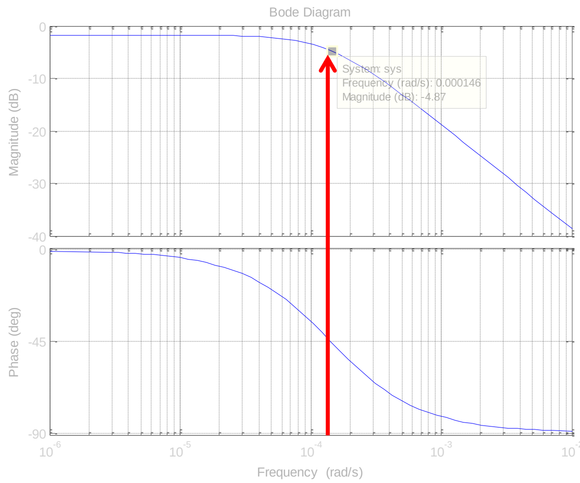
MatLAB Code

```
% close all previous figures and clear all variables
close all;
clear all;
% input the integral time constant
Tl=input('enter the process time constant: ');
G=input('enter the gain of the process: ');
% construct and display the system
sys=tf(G,[Tl 1]);
sys
% plot the frequency response
bode(sys);
% construct a new figure and plot the time response
figure;
% define a range of time
t=(0:500:5*Tl);
% use it to generate a step response plot
step(sys,t);
```


Step Response of First-Order Lag Example 19-1



Frequency Response of First-Order Lag Processes

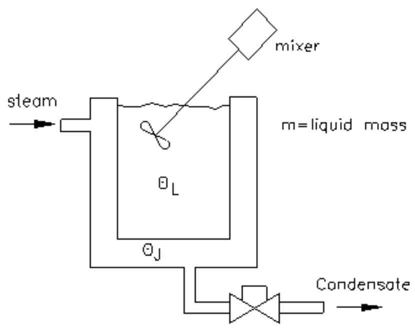


Similar to low pass filter plots.

Note the cutoff frequency, f_c is when gain is -3 dB from passband $f=f_c$

$f_c=1/\tau = 1.42\cdot10^{-4}$ rad/sec

First-Order Lag Process: Thermal Example



<u>Example 19-2:</u> Temperature of oil bath θ_L depends on the steam temperature θ_J and the thermal resistance and capacitance of the system. The equation below is the general model.

$$R_T \cdot C_T \cdot \left[\frac{d\theta_L}{dt}\right] + \theta_L = \theta_J$$

Assume constant steam flow and temperature. Theo oil-filled tank is is 1.2 m tall with a 1 m diameter. The inside film coefficient is 62 W/m²-K and the outside film coefficient is 310 W/m²-K. The tank is made of steel with a wall thickness of 1.2 cm. Find a) thermal resistance b) thermal capacitance, c) thermal time constant d) differential equation model, e) transfer function model.

Example 19-2 Solution (1)

a) Compute the thermal resistance

K_s = Thermal coefficient of Steel

$K_s = 45\ W/m\text{-}°K$

$h_i = 62\ W/m^2\text{-}°K$

$h_o = 310\ W/m^2\text{-}°K$

X = wall thickness

$X = 1.2\ cm = 0.012\ m$

Unit thermal resistance

$$R_u = \frac{1}{h_i} + \frac{X}{K_s} + \frac{1}{h_o}$$

$$R_u = \frac{1}{62\ W/m^2\text{-}°K} + \frac{0.012\ m}{45\ W/m\text{-}°K} + \frac{1}{310\ W/m^2\text{-}°K}$$

$$R_u = (0.0163 + 2.667\times10^{-4} + 0.00323)\ \frac{m^2\text{-}°K}{W} \qquad R_u = 0.01962\ \frac{m^2\text{-}°K}{W}$$

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Example 19-2 Solution (2)

Find the surface area of the tank. Assume a circular tank.

Define: A_1 = area of tank bottom
A_2 = area of tank sides

$$A_1 = \frac{\pi d_T^2}{4} = \frac{\pi (1m)^2}{4}$$

$$A_1 = 0.7854\ m^2$$

Area of sides is area of a cylinder $A_2 = \pi d_T h$

$h = 1.2m \quad d_T = 1m$

$$A_2 = \pi(1m)(1.2m) = 3.769\ m^2 \qquad A_T = A_1 + A_2$$

$$A_T = 0.7854\ m^2 + 3.769\ m^2$$

$$A_T = 4.555\ m^2$$

Example 19-2 Solution (3)

Compute total thermal resistance

$$R_T = \frac{R_u}{A} = \frac{0.01962\ m^2\text{-}^\circ K/w}{4.555\ m^2} = \boxed{0.004307\ K/w}$$ Ans

b) Thermal capacitance $C_T = mS_h$

m = mass of oil
find from density
and tank volume

S_h = specific heat of oil
$S_h = 2180\ J/kg\text{-}^\circ K$

S_h found in Appendix A

$$V = \frac{\pi d_T^2}{4}(h) = \frac{\pi (1m)^2}{4}(1.2m)$$
$$V = 0.9425\ m^3$$

$$m = \rho V = 880\ kg/m^3(0.9425\ m^3)$$
$$m = 829.4\ kg$$

Example 19-2 Solution (4)

$$C_T = (829.4\ kg)(2180\ J/kg\text{-}^\circ K)$$
$$\boxed{C_T = 1.808 \times 10^6\ J/^\circ K}$$ Ans

Use the values of R_T and C_T to find time constant

$$\tau = R_T C_T = (0.004307\ K/w)(1.808\times10^6\ J/K)$$
$$\tau = 7787\ s\ \ (129.8\ min)$$

Step change in input will take 5τ to reach final value.

5τ = 649 minutes (10.82 hours)

Example 19-2 Solution (5)

d) Find the time function

$$\tau \frac{dy}{dt} + y = Gx$$

$x = \theta_J$ Temp of Jacket

$y = \theta_L$ Temp of Liquid

G=1 in this case so:

$$7787 \frac{d\theta_L}{dt} + \theta_L = \theta_J$$

Ans

e) Find the transfer function

$$\frac{\theta_L(s)}{\theta_J(s)} = \frac{1}{1 + \tau s}$$

$$\frac{\theta_L(s)}{\theta_J(s)} = \frac{1}{1 + 7787s}$$

Ans

Dead-Time Process

Characteristic: Energy or mass transported over a distance
Common in process industries (Chemicals Refining etc)

Time domain equation:

$$f_o(t) = f_i(t - t_d)$$

$$t_d = \frac{D}{v}$$

Transfer function:

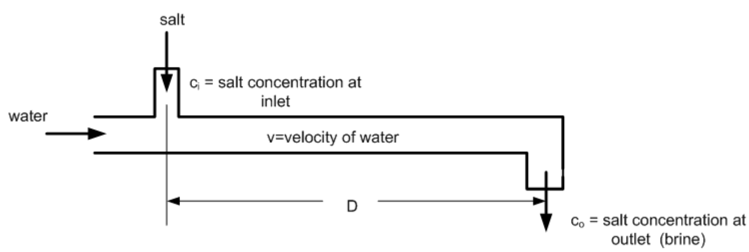
$$\frac{F_o(s)}{F_i(s)} = e^{-t_d \cdot s}$$

Where:

$f_o(t)$ = output function
$f_i(t)$ = input function
v = velocity of response travel (m/sec)
D = distance from input to output
t_d = dead-time lag (sec or minutes)
$F_o(s)$ = Laplace transform of output
$F_i(s)$= Laplace transform of input

Dead-Time Process

Example: 19-3: Determine the dead-time lag and the process transfer function if the salt-water solution travels at 0.85 m/sec and the distance to the bend is 15 m. Plot the time and frequency response of this system to a step-change in inlet concentration.



Example 19-3 Solution (1)

Define parameters

$$v = 0.85\ \text{m/sec}$$
$$D = 15\ \text{m}$$

Compute time delay

$$t_d = \frac{D}{v} = \frac{15\ \text{m}}{0.85\ \text{m/sec}} = 17.65\ \text{sec}$$

Time function:

$$c_o(t) = c_i(t - t_d)$$
$$c_o(t) = c_i(t - 17.65)$$

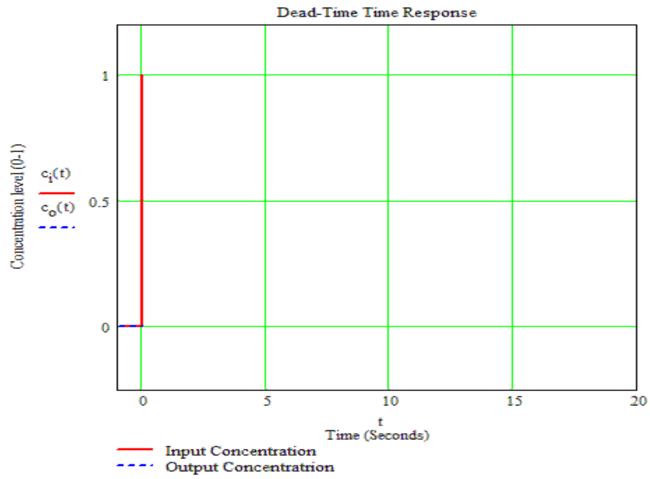
Transfer function:

$$\frac{C_o(s)}{C_i(s)} = e^{-17.65s} \cdot C(s)$$

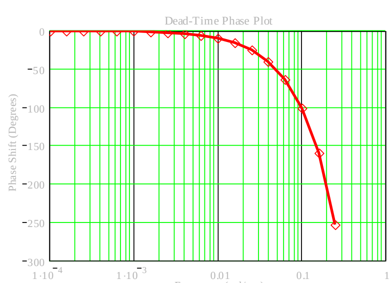
C(s) is the function describing salt concentration

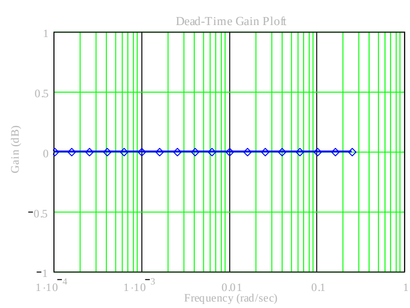
Now plot the time and frequency responses

Dead-Time Process Time Plot



Dead-Time Frequency Bode Plot





Phase increases as frequency increases. Becomes very large for high frequencies.

Gain is constant over all frequencies (0 dB G=1)

End Lesson 19: Process Characteristics-1st Order and Dead-Time Processes

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